International Journal of Theoretical Physics, Vol. 29, No. 12, 1990

# $ISO(3, 1 | N)$ and $OSp(N | 4)$ Supergravities

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*Received October 10, 1988*

We advance an $ISO(3, 1 | N)$ extended Poincaré supergravity and an $OSp(N | 4)$ de Sitter supergravity by using the supergauge action mechanisms of supergroups on the superspaces and by treating the gravitational parts of these two supergravities as the gauge theories of gravity, give a new matrix representation of $ISO(3, 1 | N)$ generators and a new one of $OSp(N | 4)$ ones, obtain the commutation and anticommutation relations of $iso(3, 1 | N)$ and $osp(N | 4)$ superalgebras, construct the actions of these supergravities and discuss some other problems. A particle multiplets method based on the supersymmetry transformation is used and the probable numbers of particles of different helicities in the two supergravities are given.

## 1. INTRODUCTION

Gravitational masses and particles exist in space-time. We treat space-time, gravitational masses, and particles as a superunified physical system, which possesses three kinds of gauge symmetries, space-time symmetries, internal symmetries (Langacker, 1981), and supersymmetry (Sohnius, 1985). These symmetries of the superunified system are represented by the gauge supergroup $G$. Let us denote the space-time manifold by $M$; thus, $\forall X \in M$, $\exists$ a superspace spanned by a set of physical (or mathematical) quantities under $G$. We denote the superspace, (Wess and Zumino, 1978) by $U$; $U$ can be chosen as

$$U \equiv V \oplus_G \Theta$$

where $V$ is the associated Bose space for the gauge action of the gauge theory of gravity (Changgui and Bangqing, 1985), and $\Theta$ is the Fermi space of dimension $N \times L$.

If we take $U$ as a supervector space, then a supervector field $X$ will

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0020-7748/90/1200-1299$06.00/0 

exist and $X$ can be written as

$$X^A = (\xi^a, \theta^i_\alpha)$$

where $X^A \in U$, $\xi^a \in V$ $(a = 1, \ldots, l)$, $\theta^i_\alpha \in \Theta$ $(\alpha = 1, \ldots, L;\ i = 1, \ldots, N)$. Here $i$ are the internal symmetry indices; $i$ will supply some irreducible representations to the group of grand unified theories; the $\alpha$ are usually taken as Majorana spinor indices. Then $\alpha = 1, \ldots, 4$. The $a$ are the gauge transformation indices of space-time. The gauge transformations of supergroup $G$ can be realized by the local action of $G$ on $X^A$. The local geometric invariant quantities under the gauge transformations can be chosen as the unified gauge action; in this way we can obtain some superunified theories.

We take $G$ as $ISO(3,1|N)$, $OSp(N|4)$, and $SU(2,2|N)$, respectively; the corresponding superspaces are $E_{(3,1)} \bigoplus_{ISO(3,1|N)} \Theta$, $E_{(3,2)} \bigoplus_{OSP(N|4)} \Theta$, and $E_{(4,2)} \bigoplus_{SU(2,2|N)} \Theta$, respectively. Thus, the Poincaré, de Sitter, and the conformal extended supergravities (van Nieuwenhuizen, 1981) can be obtained. In this paper we mainly discuss the first two. One may refer to Changgui and Bangqing (1986) for the conformal extended supergravity.

## 2. AN ACTION OF $ISO(3,1|N)$ EXTENDED POINCARÉ SUPERGRAVITY

In this supergravity (van Nieuwenhuizen *et al.*, 1978) the space-time transformation group $ISO(3,1)$ and the internal transformation group $SO(N)$ $(N > 1)$ both are subgroups of supergroup $ISO(3,1|N)$. We denote the generators of superalgebra $iso(3,1|N)$ as

$$\tau_{AB} = (M_{ab}, P_a, E_i, H^i_\alpha)$$

representations of which are chosen as

$$M_{ab} = \left[\begin{array}{c|c} -i\sigma_{ab} & 0 \\ \hline 0 & 0 \end{array}\right], \qquad P_a = \left[\begin{array}{c|c} R\gamma_a & 0 \\ \hline 0 & 0 \end{array}\right], \qquad E_i = \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & g_i \end{array}\right]$$

$$H^i_\alpha = \left[\begin{array}{c|c} 0 & 0 iR_{(i)\alpha} 0 \\ \hline \begin{matrix} 0 \\ i(CL)_{\alpha(i)} \\ 0 \end{matrix} & 0 \end{array}\right]$$

$$= \left[\begin{array}{c|c} 0 & \begin{matrix} 0 & \begin{matrix} \frac{1}{2}(1+\gamma_5)_{1\alpha} \\ \vdots \\ \frac{1}{2}(1+\gamma_5)_{4\alpha} \end{matrix} & 0 \end{matrix} \\ \hline \begin{matrix} 0 \\ \frac{1}{2}[C(1-\gamma_5)]_{\alpha 1} \cdots \frac{1}{2}[C(1-\gamma_5)]_{\alpha 4} \\ 0 \end{matrix} & 0 \end{array}\right]$$

Here

$$(\gamma_a, \gamma_b) = 2\eta_{ab}, \qquad \eta_{ab} = \mathrm{diag}(1, 1, 1, -1)$$

$$\gamma_5 = i\gamma_1\gamma_2\gamma_3\gamma_0, \qquad \sigma_{ab} = \tfrac{1}{4}[\gamma_a, \gamma_b], \qquad R = \tfrac{1}{2}(1+\gamma_5)$$

the $g_i$ are $SU(N)$ generators, $C = \gamma_0$ is the charge conjugate matrix, and $(i)$ runs over 1, 2, 3, 4, columns (rows) at each located row (column).

We find the superalgebra relations for $iso(3, 1|N)$ as follows:

$$\left.\begin{matrix}[M_{ab}, M_{cd}]\\ [M_{ab}, P_c]\end{matrix}\right\} \Rightarrow iso(3, 1), \qquad [H^i_\alpha, M_{ab}] = -i(\sigma_{ab})_{\alpha\beta}H^i_\beta$$

$$[E_i, E_j] = f^k_{ij}E_k, \qquad [H^i_\alpha, E_j] = (g_j)^{ik}H^k_\alpha$$

$$\{H^i_\alpha, H^j_\beta\} = -\tfrac{1}{2}\delta^{ij}(\gamma^a C)_{\alpha\beta}P_a$$

Now we introduce the supergauge potential

$$B^{AB}_\mu = (B^{ab}_\mu, V^a_\mu, E^i_\mu, \bar{\Lambda}^{i\alpha}_\mu)$$

corresponding to the $ISO(3, 1|N)$ supergroup generators

$$\tau_{AB} = (M_{ab}, P_a, E_i, H_{i\alpha})$$

respectively, into our theory. Defining the supergauge covariant derivative

$$D_\mu = \partial_\mu + B^{AB}_\mu \tau_{AB}$$

we get the formula of supergauge field strength $R^{AB}_{\mu\nu}$ as

$$R^{AB}_{\mu\nu} = \partial_\mu B^{AB}_\nu - \partial_\nu B^{AB}_\mu + F^{AB}_{CD,EF}B^{CD}_\mu B^{EF}_\nu$$

where $F^{AB}_{CD,EF}$ are the structure constants of supergroup $ISO(3, 1|N)$. Thus, we can write the supergauge field strength components corresponding to the generators $\tau_{AB}$ as follows:

$$R^{ab}_{\mu\nu}(M) = F^{ab}_{\mu\nu}$$

$$R^i_{\mu\nu}(E) = \partial_\mu E^i_\nu - \partial_\nu E^i_\mu + f^i_{jk}E^j_\mu E^k_\nu$$

$$R^a_{\mu\nu}(P) = J^a_{\mu\nu} + \tfrac{1}{2}(\bar{\Lambda}_{\mu\nu}\gamma^a\Lambda_{\nu\mu} + \mu \leftrightarrow \nu)$$

$$R_{\mu\nu i}(H) = \bar{\Lambda}_{\nu i}\bar{D}'_\mu + E^k_\mu(g_k)_{ij}\bar{\Lambda}_{\nu j} - \mu \leftrightarrow \nu$$

where

$$F^{ab}_{\mu\nu} = \partial_\mu B^{ab}_\nu + B^a_{\mu c}B^{cb}_\nu - \mu \leftrightarrow \nu$$

$$J^a_{\mu\nu} = \partial_\mu V^a_\nu + B^a_{\mu b}V^b_\nu - \mu \leftrightarrow \nu$$

$$D'_\mu = \partial_\mu - B_{\mu ab}\sigma^{ab}$$

A Yang-Mills type of Lagrangian of this system can be constructed as

$$L_{\text{gauge}}=-\tfrac{1}{4}\{R^{ab}_{\mu\nu}(M)R^{\mu\nu}_{ab}(M)+R^{a}_{\mu\nu}(P)R^{\mu\nu}_{a}(P)+R^{i}_{\mu\nu}(E)R^{\mu\nu}_{i}(E)$$
$$+R_{\mu\nu i}(H)CR^{\mu\nu i}(H)\}$$

where $a>b$.

We introduce the invariant geometric quantity $\dot{R}$, the curvature scalar of the gauge action space (Changgui and Bangqing, 1985) $E_{(3,1)}$, into the theory; the Lagrangian of the system can be written as

$$L=\dot{R}+L_{\text{gauge}}$$

Then the action of this system is

$$S=\int L\sqrt{-g}\,d^4x$$

## 3. SUPERSYMMETRY PARTICLE MULTIPLETS IN *ISO*(3, 1|*N*) SG

We simply denote all helicity operators by the symbol $\hat{\mathbf{J}}$. In the superalgebra $iso(3,1|N)$ the relations

$$[H^{i}_{\alpha}, M_{ab}]=-i(\sigma_{ab})_{\alpha\beta}H^{i}_{\beta}$$

are a spinor representation of the Lorentz group $SO(3,1)$; the $\sigma_{ab}$ are generators of the spinor representation. Since the space components of $M_{ab}$ represent the spin operators in the spinor representation, we have the relations

$$[H^{i}_{\alpha},\hat{\mathbf{J}}]=i(\sigma_K)_{\alpha\beta}H^{i}_{\beta} \tag{1}$$

Choose a suitable representation; from (1) we can obtain

$$[H^{i}_{\alpha},\hat{\mathbf{J}}]=\pm\tfrac{1}{2}H^{i}_{\alpha}\quad\begin{cases}\alpha=2,4\\ \alpha=1,3\end{cases} \tag{2}$$

Now we introduce the eigenstate equation

$$\hat{\mathbf{J}}|J\rangle=J|J\rangle \tag{3}$$

where $J=0,\pm\frac{1}{2},\pm1,\pm\frac{3}{2},\pm2,\pm\frac{5}{2},\pm3,\ldots$ are eigenvalues of $\hat{\mathbf{J}}$; from (2) and (3) we have following eigenequations:

$$\hat{\mathbf{J}}H^{i}_{\tau}|J\rangle=(J\pm\tfrac{1}{2})H^{i}_{\tau}|J\rangle\quad\begin{cases}\tau=1,3\\ \tau=2,4\end{cases}$$
$$\hat{\mathbf{J}}H^{i}_{\tau}H^{j}_{\tau'}|J\rangle=(J\pm1)H^{i}_{\tau}H^{j}_{\tau'}|J\rangle\quad\begin{cases}\tau,\tau'=1,3\\ \tau,\tau'=2,4\end{cases}$$
$$\hat{\mathbf{J}}H^{i}_{\tau}H^{j}_{\tau'}H^{K}_{\tau''}|J\rangle=(J\pm\tfrac{3}{2})H^{i}_{\tau}H^{j}_{\tau'}H^{k}_{\tau''}|J\rangle\quad\begin{cases}\tau,\tau',\tau''=1,3\\ \tau,\tau',\tau''=2,4\end{cases} \tag{4}$$

From the above eigenequations we know that $H_1^i$, $H_3^i$ are the operators raising helicity $\frac{1}{2}$ and $H_2^i$, $H_4^i$ are the operators lowering helicity $\frac{1}{2}$. We take a frame associated with the discussed particle; the anticommutation relations in the superalgebra $iso(3, 1|N)$ in this frame become

$$\{H_\alpha^i, H_\beta^j\} = \tfrac{1}{2}\delta^{ij}(\gamma^0 C)_{\alpha\beta} P_0 \tag{5}$$

where $P_0$ is the momentum component of the particle,

$$[(\gamma^0 C)_{\alpha\beta}] = \begin{bmatrix} 0 & & & -1 \\ & & -1 & \\ & -1 & & \\ 1 & & & 0 \end{bmatrix}$$

If we set $H_3^i = -\bar{H}_2^i$, $H_4^i = \bar{H}_1^i$, then expression (5) can be written as

$$\{H_\sigma^i, H_\rho^i\} = 0, \qquad \{\bar{H}_\sigma^i, \bar{H}_\rho^i\} = 0, \qquad \{\bar{H}_\sigma^j, H_\rho^j\} = i\frac{P_0}{2}\delta_{\sigma\rho} \tag{6}$$

Here $\sigma, \rho = 1, 2$; $H_\sigma^i$, $\bar{H}_\rho^i$ are both two-component spinors. From the first two expressions of (6) we can obtain

$$H_\sigma^i H_\sigma^i = \bar{H}_\rho^i \bar{H}_\rho^i = 0 \tag{7}$$

Thus, we conclude that if the same supersymmetry transformation is carried out successively twice, we do not obtain a new state. Therefore, the sum total of the eigenstates of distinct helicities is not larger than $N$ and the maximum of helicity will be constrained.

For example, we take $H_1^i$ and $H_2^i$ as the representatives and discuss the multiplicities of helicity multiplets. For this purpose the maximum and the minimum spin states are defined as

$$\hat{\mathbf{J}}|J\rangle = J_{\max}|J\rangle \tag{8}$$

and

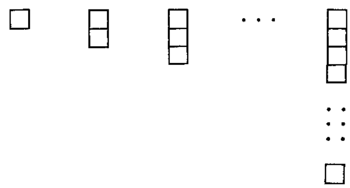
$$\hat{\mathbf{J}}|J\rangle = J_{\min}|J\rangle \tag{9}$$

where $J_{\max} = N/4$, $J_{\min} = -N/4$. Evidently, if we let $H_2^i$ $(i = 1, \ldots N)$ act on expression (8) successively, a series of multiplets may be obtained by using expression (2). The helicities of these multiplets are $N/4$, $N/4 - \frac{1}{2}, \ldots,$ $0, \ldots, -N/4 + \frac{1}{2}$, $-N/4$, respectively.

Similarly, if $H_1^i$ $(i = 1, \ldots, N)$ acts on expression (9) successively, we obtain a series of multiplets, the helicities of which are $-N/4$, $-N/4 + \frac{1}{2}, \ldots, 0, \ldots, N/4 - \frac{1}{2}$, $N/4$, respectively. A state whose helicity is higher than $N/4$ or lower than $-N/4$ cannot be obtained because of expression (7). The maximal spin state is usually considered as a one-particle state. If each $H_2^i$ $(i = 1, \ldots, N)$ acts on expression (8) once respectively, we obtain the multiplet, the spin of which is $J_{\max}-\frac{1}{2}$ and its multiplicity is $C_1^N$. If each $H_2^i$ ($i=1,\ldots,N$) acts on expression (8) twice respectively, the spin obtained is $J_{\max}-1$ and its multiplicity is $C_2^N$. In this way we can obtain a series of multiplets.

The multiplicity of the multiplet whose helicity is $J_{\max}-\frac{1}{2}$ is $C_1^N$, which is the same as the dimensionality of the antisymmetric tensor representation of the gauge group of grand unified theory $SO(N)$. When other characteristic numbers $N'$ ($N'\leq N$) are considered, a series of supergravity theories corresponding to every $N'$ can be obtained. In these theories the multiplicity of the multiplet whose helicity is $J_{\max}-\frac{1}{2}$ is also $C_1^N$. By using the Young diagrams, the multiplicities whose characteristic numbers are $N'$ ($N'\leq N$) and helicity is $J_{\max}-t/2$ ($t=0, 1,\ldots, N'$) are as follows:

$t=$ 0 1 2 3 ⋯ $N'$



$C_0^{N'}$ $C_1^{N'}$ $C_2^{N'}$ $C_3^{N'}$ ⋯ $C_N^{N'}$

Here dimensionality $C_t^{N'}=N'/t!\,(N'-t)$, $N'\leq N$.

We put suitable particles in the multiplets; then the particle multiplets in the relevant supergravity can be obtained. For example, we take the cases where $N=8$ and $N=10$; the particles in the relevant supergravities are listed in Tables I and II.

From the third expression of (6) we have

$$\{H_1^i, H_4^i\}=\{H_4^i, H_1^i\}=\frac{i}{2}P, \qquad \{H_2^i, H_3^i\}=\{H_3^i, H_2^i\}=-\frac{i}{2}P \quad (10)$$

**Table I**

| | Number of particles | | | | | |
|---|---|---|---|---|---|---|
| $N'$ | $j=2$ | $j=\frac{3}{2}$ | $j=1$ | $j=\frac{1}{2}$ | $j=0$ | $j=-\frac{1}{2}$ |
| 1 | $C_0^1=1$ | $C_1^1=1$ | | | | |
| 2 | $C_0^2=1$ | $C_1^2=2$ | $C_2^2=1$ | | | |
| 3 | $C_0^3=1$ | $C_1^3=3$ | $C_2^3=3$ | $C_3^3=1$ | | $C_3^3=1$ |
| 4 | $C_0^4=1$ | $C_1^4=4$ | $C_2^4=6$ | $C_3^4=4$ | $2C_4^4=2$ | $C_3^4=4$ |
| 5 | $C_0^5=1$ | $C_1^5=5$ | $C_2^5=10$ | $C_3^5+C_5^5=11$ | $2C_4^5=10$ | $C_5^5+C_3^5=11$ |
| 6 | $C_0^6=1$ | $C_1^6=6$ | $C_2^6+C_6^6=16$ | $C_3^6+C_5^6=26$ | $2C_4^6=30$ | $C_3^6+C_5^6=26$ |
| 7 | $C_0^7=1$ | $C_1^7+C_7^7=8$ | $C_2^7+C_6^7=28$ | $C_3^7+C_5^7=56$ | $2C_4^7=70$ | $C_5^7+C_3^7=56$ |
| 8 | $C_0^8=1$ | $C_1^8=8$ | $C_2^8=28$ | $C_3^8=56$ | $C_4^8=70$ | $C_5^8=56$ |

**Table II**

| $N'$ | Number of particles $J=\frac{5}{2}$ | $J=2$ | $J=\frac{3}{2}$ | $J=1$ | $J=\frac{1}{2}$ | $J=0$ | $J=-\frac{1}{2}$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | | | | |
| 2 | 1 | 2 | 1 | | | | |
| 3 | 1 | 3 | 3 | 1 | | | |
| 4 | 1 | 4 | 6 | 4 | 1 | | 1 |
| 5 | 1 | 5 | 10 | 10 | 5 | 2 | 5 |
| 6 | 1 | 6 | 15 | 20 | 16 | 12 | 16 |
| 7 | 1 | 7 | 21 | 36 | 42 | 42 | 42 |
| 8 | 1 | 8 | 29 | 64 | 98 | 112 | 98 |
| 9 | 1 | 10 | 45 | 120 | 210 | 252 | 210 |
| 10 | 1 | 10 | 45 | 120 | 210 | 252 | 210 |

From the above expressions, we know that an operator raising helicity $\frac{1}{2}$ and an operator lowering helicity $\frac{1}{2}$ act successively and respectively in different sequences on a state; we can obtain two states, the difference of which in space-time is $\pm(i/2)P_0$. In the associated frame, $\pm(i/2)P_\alpha$ becomes $\pm(i/2)P_0$. If a particle exists in an original state, by a helicity state transformation, the position of the particle will undergo a translation.

## 4. AN ACTION OF *OSp*(*N*|4) EXTENDED DE SITTER SG

The de Sitter space-time transformation group $DS(3,2)$ and the internal transformation group $SO(N)$ are both subgroups of supergroup $OSp(N|4)$ in this SG (West and Stelle, 1979). We denote generators of superalgebra $OSp(N|4)$ as $\hat{\tau}_{AB}=(M_{ab}, \hat{P}_a, E_i, O^i_\alpha)$; a representation of these generators can be found as

$$M_{ab}=\left[\begin{array}{c|c} -i\sigma_{ab} & 0 \\ \hline 0 & 0 \end{array}\right], \qquad \hat{P}_a=\left[\begin{array}{c|c} 1/2\gamma_a & 0 \\ \hline 0 & 0 \end{array}\right], \qquad E_i=\left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & g_i \end{array}\right]$$

$$O^i_\alpha=\left[\begin{array}{c|c} 0 & 0 \quad i\delta_{(i)\alpha} \quad 0 \\ \hline \begin{array}{c} 0 \\ iC_{\alpha(i)} \\ 0 \end{array} & 0 \end{array}\right]=\left[\begin{array}{c|c} 0 & 0 \quad \begin{array}{c} i\delta_{1\alpha} \\ \vdots \\ i\delta_{4\alpha} \end{array} \quad 0 \\ \hline \begin{array}{c} 0 \\ iC_{\alpha 1}\ldots iC_{\alpha 4} \\ 0 \end{array} & 0 \end{array}\right]$$

and we find that the nonzero superalgebra relations of $\hat{\tau}_{AB}$ are

$$[M_{ab}, M_{cd}] = -i\eta_{ad}M_{bc} + \cdots, \qquad [M_{ab}, \hat{P}_c] = i\eta_{ac}\hat{P}_b - i\eta_{bc}\hat{P}_a$$

$$[O^i_2, M_{ab}] = -i(\sigma_{ab})_{\alpha\beta}O^i_\beta, \qquad [O^i_\alpha, \hat{P}_a] = -\frac{i}{2}(\gamma^T_a)_{\alpha\beta}O^i_\beta$$

$$[\hat{P}_a, \hat{P}_b] = -M_{ab}, \qquad [E_i, E_j] = f^K_{ij}E_K, \qquad [O^i_\alpha, E_K] = (g_K)^{ij}O^i_\alpha$$

$$\{O^i_\alpha, O^j_\beta\} = i\delta^{ij}\{2(C\sigma^{ab})_{\alpha\beta}M_{ab} + (\gamma^a C)_{\alpha\beta}\hat{P}_a\}$$

Here the subalgebra $ds(3,2)$ of the algebra $OS_P(N|4)$ is constructed from the first, second, and fifth expressions. Let

$$\hat{B}^{AB}_{\mu\nu} = (B^{ab}_\mu, \hat{V}^a_\mu, E^i_\mu, \bar{\Sigma}^{i\alpha}_\mu)$$

be the supergauge potential corresponding to the generators

$$\hat{\tau}_{AB} = (M_{ab}, \hat{P}_a, E_i, O_{i\alpha})$$

respectively; then we have the supergauge field strength

$$\hat{R}^{AB}_{\mu\nu} = \partial_\mu B^{AB}_\nu - \partial_\nu B^{AB}_\mu + \hat{F}^{AB}_{CD,EF}\hat{B}^{CD}_\mu \hat{B}^{EF}_\nu$$

where $\hat{F}^{AB}_{CD,EF}$ are structure constants of supergroup $OSp(N|4)$. Thus, we get

$$\hat{R}^{ab}_{\mu\nu}(M) = \hat{R}^{ab}_{\mu\nu}(M)_{\text{gra}} + \hat{R}^{ab}_{\mu\nu}(M)_{\text{sup}} = F^{ab}_{\mu\nu} + \hat{V}^{ab}_{\mu\nu} - 2\bar{\Sigma}_\mu(C\sigma^{ab}C^{-1})\Sigma_\nu$$

$$\hat{R}^a_{\mu\nu}(\hat{P}) = \hat{R}^a_{\mu\nu}(P)_{\text{gra}} + \hat{R}^a_{\mu\nu}(P)_{\text{sup}} = -\hat{J}^a_{\mu\nu} - (\bar{\Sigma}_{\mu i}\gamma^a\Sigma_{\nu i} + \mu \leftrightarrow \nu)$$

$$\hat{R}_{\mu\nu i}0[\partial\mu] = \bar{\Sigma}_{\nu i}\bar{D}'_\mu + \frac{1}{2}\bar{\Sigma}_{\nu i}\hat{\gamma}_\mu + E^K_\mu(g_K)_{ij}\Sigma_{\nu j} - \mu \leftrightarrow \nu$$

$$\hat{R}^K_{\mu\nu}(E) = \partial_\mu E^K_\nu - \partial E^K_\mu + f^K_{ij}E^i_\mu E^j_\nu$$

where

$$\hat{V}^{ab}_{\mu\nu} = \hat{V}^a_\mu \hat{V}^b_\nu - \hat{V}^a_\nu \hat{V}^b_\mu$$

$$\hat{\gamma}_\mu = (\gamma^T_a)\hat{V}^a_\mu$$

$$\hat{J}^a_{\mu\nu} = \partial_\mu \hat{V}^a_\nu + B^a_{\mu b}\hat{V}_\nu - \mu \leftrightarrow \nu$$

A Yang–Mills type of Lagrangian of the de Sitter SG can be constructed as

$$\begin{aligned}\hat{L} &= -\tfrac{1}{4}St_r(\hat{R}_{\mu\nu}\hat{R}^{\mu\nu})\\ &= -\tfrac{1}{4}\{\hat{R}^{ab}_{\mu\nu}(M)\hat{R}^{\mu\nu}_{ab}(M) + \hat{R}^a_{\mu\nu}(\hat{P})\hat{R}^{\mu\nu}_a(P)\\ &\quad + R^i_{\mu\nu}(E)\hat{R}^{\mu\nu}_i(E) + \hat{R}_{\mu\nu i}0[\partial\mu]C\hat{R}^{\mu\nu i}0[\partial\mu]\} \qquad (a > b)\end{aligned}$$

We can write $\hat{L}$ in the form

$$\hat{L}=\hat{L}_{\text{gra}}+\hat{L}_{\text{sup}}+\hat{L}_{\text{int}}+\hat{L}_{\text{gra-sup}}$$

where

$$\hat{L}_{\text{gra}}=-\tfrac{1}{4}F^{ab}_{\mu\nu}F^{\mu\nu}_{ab}-\tfrac{1}{4}\hat{V}^{ab}_{\mu\nu}\hat{V}^{\mu\nu}_{ab}-\tfrac{1}{4}\hat{J}^{a}_{\mu\nu}\hat{J}^{\mu\nu}_{a}-\tfrac{1}{2}\hat{F}^{ab}_{\mu\nu}\hat{V}^{\mu\nu}_{ab}$$

$$\hat{L}_{\text{sup}}=-\tfrac{1}{4}\hat{R}_{\mu\nu i}0[\partial\mu]C\hat{R}^{\mu\nu i}0[\partial\mu]-\tfrac{1}{4}\hat{R}^{ab}_{\mu\nu}(M)_{\text{sup}}\hat{R}^{\mu\nu}_{ab}(M)_{\text{sup}}$$
$$-\tfrac{1}{4}\hat{R}^{a}_{\mu\nu}(\hat{P})_{\text{sup}}\hat{R}^{\mu\nu}_{a}(\hat{P})_{\text{sup}}$$

$$\hat{L}_{\text{int}}=-\tfrac{1}{4}\hat{R}^{i}_{\mu\nu}(E)\hat{R}^{\mu\nu}_{i}(E)$$

$$\hat{L}_{\text{gra-sup}}=-\tfrac{1}{2}\hat{R}^{ab}_{\mu\nu}(M)_{\text{gra}}\hat{R}^{\mu\nu}_{ab}(M)_{\text{sup}}-\tfrac{1}{2}\hat{R}^{a}_{\mu\nu}(\hat{P})_{\text{gra}}\hat{R}^{\mu\nu}_{a}(\hat{P})_{\text{sup}}$$

Here $\hat{L}_{\text{gra}}$ is the Lagrangian of de Sitter gravity (Changgui *et al.*, 1990), in which the first two terms are Einstein and cosmology terms of de Sitter gravity, respectively; thereby the de Sitter gauge theory of gravity and general relativity can be contained in the SG. The action of this SG has the form

$$\hat{S}=\int \hat{L}\sqrt{-g}\,d^4x$$

## 5. NUMBERS OF PARTICLES IN *OSp*(*N*|4) SUPERGRAVITY

The space-time discussed in this SG is the de Sitter one; the particles described in the SG are moving in a de Sitter sphere which possesses constant curvature. We still take a frame associated with the particle discussed. In this frame the anticommutation relation of the algebra $OSp(N|4)$ becomes

$$\{O^i_\alpha, O^j_\beta\}=\delta^{ij}\{2(C\sigma^{\mu\nu})_{\alpha\beta}M_{\mu\nu}-(\gamma^0C)_{\alpha\beta}P_0\} \tag{11}$$

where $u$, $v=1, 2, 3$. From the rhs of expression (11), we get

$$\{O^i_\alpha, O^i_\alpha\}=0, \qquad \{O^i_1, O^i_2\}=0, \qquad \{O^i_3, O^i_4\}=0 \tag{12}$$

In expression (6) the above relations are included, but the relation

$$\{O^i_\lambda, O^i_{\lambda'}\}\neq 0, \qquad (\lambda=1, \lambda'=3 \quad \text{or} \quad \lambda=2, \lambda'=4) \tag{13}$$

is the difference of them. From expression (11) and the last one, we know that if two operators of expression (13) act on a state in opposite sequences respectively and successively, the two states obtained will differ in an $SO(3)$ rotation in a three-dimensional space, besides in a de Sitter translation $\pm iP_0$ in de Sitter space. This is one of the differences between $OSp(N|4)$ SG and $ISO(3, 1|N)$ SG. By using the results obtained above, the method determining the number of particles in $ISO(3, 1|N)$ SG can be extended in $OSp(N|4)$

SG. But the number of probable states having the same helicity will increase in $OSp(N|4)$ SG and the differences of these states moving in space-time will become more complex in $OSp(N|4)$ SG.

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